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NUMBER AND THE FUNDAMENTAL LAWS OF ALGEBRA

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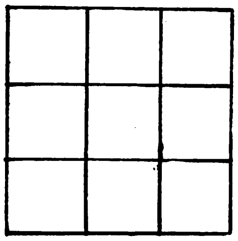
III. GRAPHICAL METHOD*

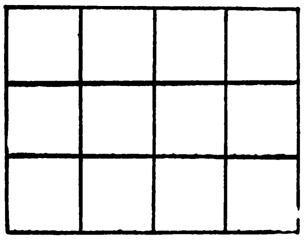
ONE OF THE SIMPLEST methods of approach to the handling of the fundamental laws of algebra is through the graph. The pupil in the grades, who has had some acquaintance with elementary geometrical principles, quickly grasps the ideas involved, through the vivid appeal of simple figures to the eye.

Certain fundamental assumptions must be made, in order to lay the foundations. The student is supposed to know how to construct a right angle, a square, and a rectangle. Moreover a knowledge of addition and subtraction is presupposed. A number is to be represented by a linear segment. The sign + indicates addition, the sign — subtraction; and it is presumed that $a - a = o$ where a may represent either a linear segment or an area.

If x be a number representing a linear segment, then $x \cdot x = x^2$ is understood to represent a given square each side of which is x. Thus the quantity $3 \times 3 = 9$ may be represented to the eye as follows:



In like manner $x \cdot y = xy$ is understood to represent a given rectangle. Thus $3 \times 4 = 12$ may be represented to the eye as follows:



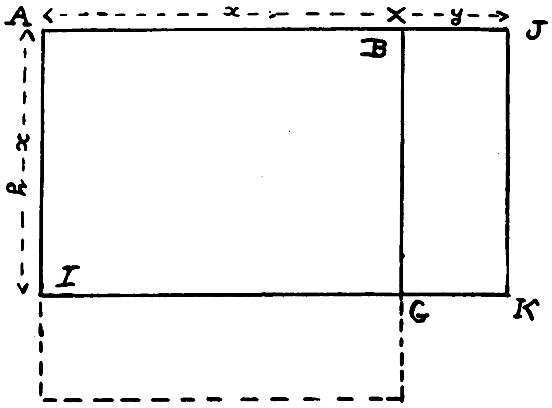
Before proceeding, it is necessary to establish the distributive law, as applied to areas, viz.:

$$(a + b)\ c = ac + bc$$

Let us assume, $x = a, y = b, x - y = c$. Then it remains to prove that

$$(x + y)\ (x - y) = x(x - y) + (x - y)$$

Now let us consider the following figure:



By inspection,

$$\text{ABGI} + \text{BGKJ} = \text{AJKI}$$
$$\therefore\ x\ (x - y) + y\ (x - y) = (x + y)\ (x - y)$$
$$\text{or}\quad ac + bc = (a + b)\ c$$

Proof of Law: $a \times - b = - ab$

By a method similar to that above, it may easily be shown that $x(x - y) = x^2 - xy$. Hence from the preceding case, we have

$$(x + y)\ (x - y) = x^2 - xy + yx - y^2$$

A figure demonstates that $x \cdot y = yx$
Accordingly,

$$(x + y)\ (x - y) = x^2 - y^2$$

In more general terms, let us attempt to prove that

$$(c + a)\ (d - b) = cd + ad - cb - ab$$

Now we know that this product can have but one of the two possible values:

$$cd + ad + cb + ab \text{............(I)}$$

or

$$cd + ad - cb - ab \text{............(II)}$$

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*For Parts I and II see THE HIGH SCHOOL JOURNAL for March and April, respectively, 1921.

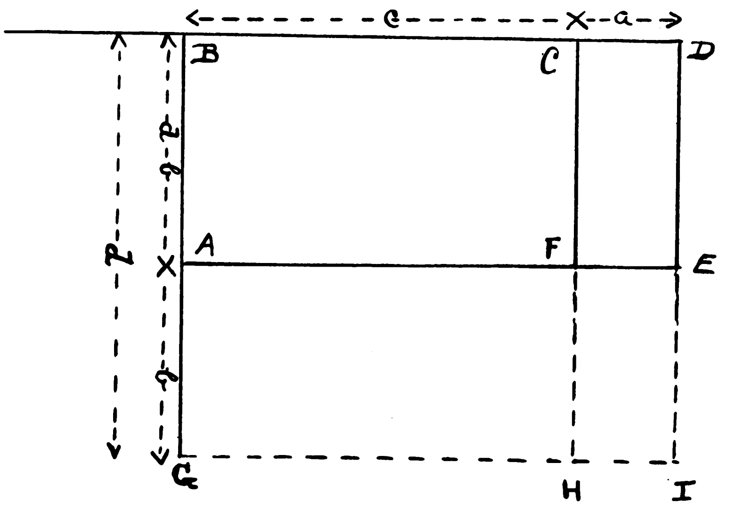
depending on whether, for example

$$c \times -b = +cb$$

or

$$c \times -b = -cb$$

Consider now the following figure:



Now by inspection

ABDE = GBCH + HCDI — GAFH — HFEI

That is,

$$(c + a)\ (d - b) = cd + ad - cb - ab$$

As this agrees with (II) above, it follows that we must have

$$c \times -b = -cb$$

Proof of Law: $-a \times -b = +ab$

The expression $(c - a)\ (d - b)$ must take one or other of the values

$$cd - cb - ad - ab \text{............(I)}$$

or

$$cd - cb - ad + ab \text{............(II)}$$

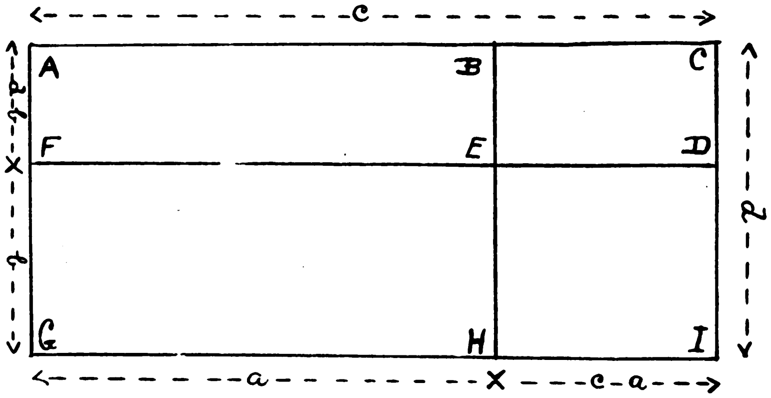
depending on whether

$$-a \times -b = -ab$$

or

$$-a \times -b = +ab$$

Consider now the following figure:



Now by inspection

EBCD = GACI — GFDI — GABH + GFEH

That is,

$$(c - a)\ (d - b) = cd - cd - ad + ab$$

As this agrees with (II) above, it follows that we must have

$$-a \times -b = +ab$$

The Principle of Continuity

In setting up proofs of the fundamental laws of algebra, resort may be had to the principle of continuity. An equation is accordingly defined to be such a relation between algebraic unknowns, connected by the sign of equality, as will remain valid for all values of the quantities involved. The further assumption is also made:

$$a \times o = o$$

By means of the equations already established above, it may readily be proved that

$$a \times -b = -ab$$

and

$$-a \times -b = +ab$$

From the two preceding articles, we have respectively:

$$(c + a)\ (d - b) = cd + ad - cb - ab \text{............(1)}$$

and

$$(c - a)\ (d - b) = cd - cb - ad + ab \text{........(2)}$$

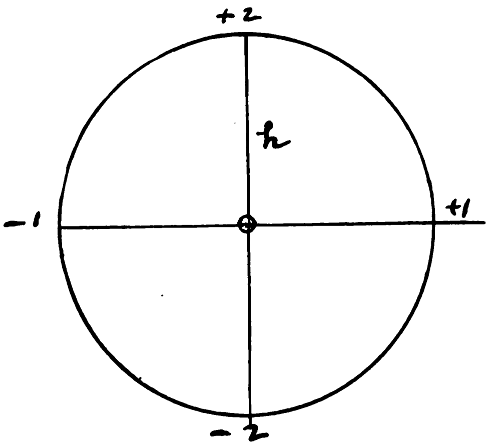
According to our assumptions, these equations hold good when $c = o$ and $d = o$. Hence we have, making use of the assumption $a \times o = o$,

$$a \times -b = -ab$$
$$-a \times -b = +ab$$

IV. THE USE OF IMAGINARIES

In order to set up the machinery for the establishment of the fundamental laws of algebra by the use of the imaginary number, we may resort to a definite physical conception of the meaning of signs and symbols. The processes prescribed follow as the natural result of generalizations drawn from familiar principles.

In geometry, we are familiar with the theorem expressed as follows: $h^2 = ab$, where h is the perpendicular dropped from any point of the circumference upon the diameter, and a and b the segments of that diameter. Let us now assume that this law holds for directed lines. Choosing a circle of unit radius with centre at the origin, we have:



$$h^2 = (+1)(-1) = -1$$

$$\therefore\ h = \pm\sqrt{-1} = \pm i$$

Now, ordinarily, we say that the result of multiplying a number by — 1 is to rotate it about the origin in a counter-clockwise direction through 180°. For this is equivalent, in the event, to reversing the line's direction. Since $i \times i = -1$, we may logically conclude that to multiply + 1 by *i*, is to rotate it counter-clockwise through an angle of 90°. By an inspection of the series $1, i, i^2, i^3, i^4, i^5, i^6, i^7$, which is equivalent to $1, i, -1, -i, 1, i, -1, -i$, we note the periodic nature of the series, each multiplication by *i* corresponding to a rotation of the unit counter-clockwise through an angle of 90°.

We may now lay down a number of definitions, which accord fully with our understanding of the meaning of the symbols involved. Thus we choose a certain distance as the unit. The letter *a*, considered as a symbol of geometrical operation, indicates that a line is to be extended or stretched to *a* times its original length. The symbol + indicates that the line is to be drawn in a certain chosen direction; and the symbol — indicates that the line to which it is applied has been turned (counter-clockwise, let us say) through two right angles.

By means of these symbols of operation each carrying a specific and unmistakable meaning, it is not difficult to establish the chief fundamental rules of Algebra, which are as follows:

$$a + b = b + a;\quad a - b = -b + a$$
$$a + (+b) = a + b;\ a + (-b) = a - b$$
$$a - (+b) = a - b;\quad a - (-b) = a + b$$
$$(+a) \times (+b) = +ab;\ (+a) \times (-b = -ab$$
$$(-a) \times (+b) = -ab;\ (-a) \times (-b) - + ab$$
$$ab = ba$$

Suffice it to show the truth of just one of the fundamental laws, in conclusion, viz.:

$$(-a) \times (-b) = +ab$$

Proof. Now (— *b*) performed on the unit carries the initial point *b* feet in the positive direction and then rotates it through two right angles.

Next, (— *a*) performed on this line *b* feet long in the negative direction extends it to *a* times its length, i. e., makes it *ab* feet, and turns it through two right angles.

Thus we see that the initial point has been carried *ab* feet from its chosen position in the positive direction; and consequently

$$(-a) \times (-b) = +ab$$

In conclusion, a mere word regarding the laws of signs. The phrase: "Best by Test," will illustrate. The operations indicated by the signs are those which experience warrants as to validity; the best rule is that which stands the test. Despite the care, in building up suitable initial assumptions, which I have exercised in this monograph, I am inclined to believe that it is simplest and most satisfactory, after all, to regard the laws of signs as the ineluctable data of truth—the rules of the game of experience.

ON LITERARY TASTE

By L. A. WILLIAMS
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MANY YEARS AGO while taking a course labelled "English 15" I learned a paragraph from one, Bacon, in which occurred the phrase, "Reading maketh a full man." I wonder what the author of that statement would add to it if he could be privileged to see and read the mass of literature (?) set forth on the book stalls of our present day railway stations in the United States.

Truly these purveyors of reading matter are making a desperate attempt to satisfy all tastes whether good, bad, or indifferent. Saucy, snappy, breezy stories; digests and current opinions; pictorial and graphic arts; work that is cosmopolitan, metropolitan, and of the world; daily, weekly, monthly, are set forth for the delectation of literary gourmandizing. If, in the days of Bacon, a man could become a full man by reading, what can be said of the verbal menu of today?

It is recognized and remembered that this same